

Tables of transforms

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Transform	Notation	
Laplace	$X(s)$	
Fourier Transform (CTFT)	$X(\omega)$ or $X(\Omega)$	<i>Signals and Systems using MATLAB</i> (Luis Chapparo)
	$G(f)$	<i>Modern Digital and Analog Communication Systems</i> (Lathi)
Discrete-Time Fourier Transform (DTFT)	$X(e^{j\omega})$	
Discrete Fourier Transform (DFT)	X_k	
Z-transform	$X(z)$	

Laplace Transform

Definition of Laplace Transform

$$X(s) = \mathcal{L}[x(t)] = \int_{-\infty}^{\infty} x(t)e^{-st} dt$$

Definition of Inverse Laplace Transform

$$x(t) = \mathcal{L}^{-1}[X(s)] = \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} X(s)e^{st} ds$$

Laplace Transform common pairs

Input $x(t)$	Output $X(s)$	ROC
$\delta(t)$	$\leftrightarrow 1$	whole s-plane
$u(t)$	$\leftrightarrow \frac{1}{s}$	$\mathcal{R}e[s] > 0$
$t \cdot u(t)$	$\leftrightarrow \frac{1}{s^2}$	$\mathcal{R}e[s] > 0$
$e^{-at}u(t), a > 0$	$\leftrightarrow \frac{1}{s+a}$	$\mathcal{R}e[s] > -a$
$\cos(\omega t)u(t)$	$\leftrightarrow \frac{s}{s^2 + \omega^2}$	$\mathcal{R}e[s] > 0$
$\sin(\omega t)u(t)$	$\leftrightarrow \frac{\omega}{s^2 + \omega^2}$	$\mathcal{R}e[s] > 0$
$e^{-at} \cos(\omega t)u(t), a > 0$	$\leftrightarrow \frac{s+a}{(s+a)^2 + \omega^2}$	$\mathcal{R}e[s] > -a$
$e^{-at} \sin(\omega t)u(t), a > 0$	$\leftrightarrow \frac{\omega}{(s+a)^2 + \omega^2}$	$\mathcal{R}e[s] > -a$
te^{-at}	$\leftrightarrow \frac{1}{(s+a)^2}$	$\mathcal{R}e[s] > -a$
$1 - e^{-at}$	$\leftrightarrow \frac{a}{s(s+a)}$	$\mathcal{R}e[s] > -a$
t^2	$\leftrightarrow \frac{2}{s^3}$	$\mathcal{R}e[s] > 0$
t^N	$\leftrightarrow \frac{N!}{s^{N+1}}$	$\mathcal{R}e[s] > 0$

Causal vs anti-causal Laplace Transforms

Causal	Anti-causal
$X_c(s) = \mathcal{L}[x_c(t)]$	$X_{ac}(s) = \mathcal{L}[x_{ac}(-t)] _{-s}$
Nothing special.	That is, to find an anti-causal Laplace transform of a signal, replace t with $-t$, compute the Laplace transform as normal, then replace every s with $-s$ in the expression.

Laplace Transform properties

Causal functions and constants	$\alpha x(t), \beta y(t)$	$\alpha X(s), \beta Y(s)$
Linearity	$\alpha x(t) + \beta y(t)$	$\alpha X(s) + \beta Y(s)$
Right shift in time	$x(t - \alpha)u(t - \alpha)$	$e^{-\alpha s} X(s)$
Frequency shifting	$e^{\alpha t} x(t)$	$X(s - \alpha)$
Convolution in the time-domain	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$
Convolution in the s-domain	$x_1(t)x_2(t)$	$X_1(s) * X_2(s)$
Multiplication by t	$tx(t)$	$-\frac{dX(s)}{ds}$
Multiplication by t^n	$t^n x(t)$	$(-1)^n \frac{d^n X(s)}{ds^n}$
Division by t	$\frac{1}{t} x(t)$	$\int_s^\infty X(u) du$
Multiplication by a Sinusoid	$x(t) \cos(\omega t)$	$\frac{1}{2} [X(s + j\omega) + X(s - j\omega)]$
	$x(t) \sin(\omega t)$	$\frac{j}{2} [X(s + j\omega) - X(s - j\omega)]$
1 st Derivative	$\frac{dx(t)}{dt}$	$sX(s) - x(0^-)$
2 nd Derivative	$\frac{d^2 x(t)}{dt^2}$	$s^2 X(s) - sx(0^-) - x'(0)$
Nth Derivative	$\frac{d^N x(t)}{dt^N}$	$s^N X(s) - \sum_{k=0}^{N-1} x^{(k)}(0^-) s^{N-1-k}$
Integral of a causal signal	$\int_{0^-}^t x(\lambda) d\lambda$	$\frac{X(s)}{s}$
Expansion/contraction	$x(\alpha t), \alpha \neq 0$	$\frac{1}{ \alpha } X\left(\frac{s}{\alpha}\right)$
Initial value theorem	$x(0) = \lim_{s \rightarrow \infty} s \cdot X(s)$	
Final value theorem	$\lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} s \cdot X(s)$	

Fourier Transform

Definition of Fourier Transform

$$\begin{aligned} X(\omega) = F[x(t)] &= \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt \\ &= \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft} dt \end{aligned}$$

Definition of Inverse Fourier Transform

$$\begin{aligned} x(t) = F^{-1}[X(\omega)] &= \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)e^{j\omega t} d\omega \\ &= \int_{-\infty}^{\infty} X(f)e^{j2\pi ft} df \end{aligned}$$

(Chaparro) Fourier Transform common pairs

Input $x(t)$	Output $X(\omega)$
$\delta(t)$	$\leftrightarrow 1$
$\delta(t - \tau)$	$\leftrightarrow e^{-j\omega\tau}$
$u(t)$	$\leftrightarrow \frac{1}{j\omega} + \pi\delta(\omega)$
$u(-t)$	$\leftrightarrow \frac{-1}{j\omega} + \pi\delta(\omega)$
$\text{sign}(t) = 2[u(t) - 0.5]$	$\leftrightarrow \frac{2}{j\omega}$
$A, -\infty < t < \infty$	$\leftrightarrow 2\pi A\delta(\omega)$
$Ae^{-at}u(t), a > 0$	$\leftrightarrow \frac{A}{j\omega + a}$
$Ate^{-at}u(t), a > 0$	$\leftrightarrow \frac{A}{(j\omega + a)^2}$
$e^{-a t }, a > 0$	$\leftrightarrow \frac{2a}{a^2 + \omega^2}$
$\cos(\omega_0 t), -\infty < t < \infty$	$\leftrightarrow \pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
$\sin(\omega_0 t), -\infty < t < \infty$	$\leftrightarrow -j\pi[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
$p(t) = A[u(t + \tau) - u(t - \tau)], \tau > 0$	$\leftrightarrow 2A\tau \frac{\sin(\omega\tau)}{(\omega\tau)}$
$\frac{\sin(\omega_0 t)}{\pi t}$	$\leftrightarrow P(\omega) = u(\omega + \omega_0) - u(\omega - \omega_0)$
$x(t)\cos(\omega_0 t)$	$\leftrightarrow 0.5[X(\omega - \omega_0) + X(\omega + \omega_0)]$

(Chaparro) Fourier Transform Properties

Signals and constants	$x(t), y(t), z(t), \alpha, \beta$	$X(\omega), Y(\omega), Z(\omega)$
Linearity	$\alpha x(t) + \beta y(t)$	$\alpha X(\omega) + \beta Y(\omega)$
Expansion/contraction in time	$x(\alpha t), \alpha \neq 0$	$\frac{1}{ \alpha } X\left(\frac{\omega}{\alpha}\right)$
Reflection	$x(-t)$	$X(-\omega)$
Parseval's energy relation	$E_x = \int_{-\infty}^{\infty} x(t) ^2 dt$	$E_x = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) ^2 d\omega$
Duality	$X(t)$	$2\pi x(-\omega)$
Time differentiation	$\frac{d^n x(t)}{dt^n}, n \geq 1$	$(j\omega)^n X(\omega)$
Frequency differentiation	$-jtx(t)$	$\frac{dX(\omega)}{d\omega}$
Integration	$\int_{-\infty}^t x(t') dt'$	$\frac{X(\omega)}{j\omega} + \pi X(0)\delta(\omega)$
Time shifting	$x(t - \alpha)$	$e^{-j\alpha\omega} X(\omega)$
Frequency shifting	$e^{j\omega_0 t} x(t)$	$X(\omega - \omega_0)$
Modulation	$x(t) \cos(\omega_c t)$	$0.5[X(\omega - \omega_c) + X(\omega + \omega_c)]$
Periodic signals	$x(t) = \sum_k X_k e^{jk\omega_0 t}$	$X(\omega) = \sum_k 2\pi X_k \delta(\omega - k\omega_0)$
Symmetry	$x(t)$ real	$ X(\omega) = X(-\omega) $ $\angle X(\omega) = -\angle X(-\omega)$
Convolution in time	$z(t) = [x^*y](t)$	$Z(\omega) = X(\omega)Y(\omega)$
Windowing/Multiplication	$x(t)y(t)$	$\frac{1}{2\pi} [X * Y](\omega)$
Cosine transform	$x(t)$ even	$X(\omega) = \int_{-\infty}^{\infty} x(t) \cos(\omega t) dt$, real
Sine transform	$x(t)$ odd	$X(\omega) = -j \int_{-\infty}^{\infty} x(t) \sin(\omega t) dt$, imaginary

(Lathi/Ding) Fourier Transform Common Pairs (Table 3.1)

	Input $g(t)$	Output $G(f)$	ROC
1	$e^{-at}u(t)$	$\leftrightarrow \frac{1}{a+j2\pi f}$	$a > 0$
2	$e^{at}u(-t)$	$\leftrightarrow \frac{1}{a-j2\pi f}$	$a > 0$
3	$e^{-a t }$	$\leftrightarrow \frac{2a}{a^2+(2\pi f)^2}$	$a > 0$
4	$te^{-at}u(t)$	$\leftrightarrow \frac{1}{(a+j2\pi f)^2}$	$a > 0$
5	$t^n e^{-at}u(t)$	$\leftrightarrow \frac{n!}{(a+j2\pi f)^{n+1}}$	$a > 0$
6	$\delta(t)$	$\leftrightarrow 1$	
7	1	$\leftrightarrow \delta(f)$	
8	$e^{j2\pi f_0 t}$	$\leftrightarrow \delta(f - f_0)$	
9	$\cos 2\pi f_0 t$	$\leftrightarrow 0.5[\delta(f + f_0) + \delta(f - f_0)]$	
10	$\sin 2\pi f_0 t$	$\leftrightarrow j0.5[\delta(f + f_0) - \delta(f - f_0)]$	
11	$u(t)$	$\leftrightarrow \frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$	
12	$\text{sgn } t$	$\leftrightarrow \frac{2}{j2\pi f}$	
13	$\cos 2\pi f_0 t u(t)$	$\leftrightarrow \frac{1}{4}[\delta(f - f_0) + \delta(f + f_0)] + \frac{j2\pi f}{(2\pi f_0)^2 - (2\pi f)^2}$	
14	$\sin 2\pi f_0 t u(t)$	$\leftrightarrow \frac{1}{4j}[\delta(f - f_0) - \delta(f + f_0)] + \frac{2\pi f_0}{(2\pi f_0)^2 - (2\pi f)^2}$	
15	$e^{-at} \sin 2\pi f_0 t u(t)$	$\leftrightarrow \frac{2\pi f_0}{(a+j2\pi f)^2 + 4\pi^2 f_0^2}$	$a > 0$
16	$e^{-at} \cos 2\pi f_0 t u(t)$	$\leftrightarrow \frac{a+j2\pi f}{(a+j2\pi f)^2 + 4\pi^2 f_0^2}$	$a > 0$
17	$\Pi\left(\frac{t}{\tau}\right)$	$\leftrightarrow \tau \text{sinc}(\pi f \tau)$	
18	$2B \text{sinc}(2\pi B t)$	$\leftrightarrow \Pi\left(\frac{f}{2B}\right)$	
19	$\Delta\left(\frac{t}{\tau}\right)$	$\leftrightarrow \frac{\tau}{2} \text{sinc}^2\left(\frac{\pi f \tau}{2}\right)$	
20	$B \text{sinc}^2(\pi B t)$	$\leftrightarrow \Delta\left(\frac{f}{2B}\right)$	
21	$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\leftrightarrow f_0 \sum_{n=-\infty}^{\infty} \delta(f - n f_0)$	$f_0 = \frac{1}{T}$
22	$e^{-t^2/2\sigma^2}$	$\leftrightarrow \sigma\sqrt{2\pi} e^{-2(\sigma\pi f)^2}$	
23	$g(t) \cos(2\pi f_0 t)$	$\leftrightarrow \frac{1}{2}[G(f + f_0) + G(f - f_0)]$	
24	$\Pi\left(\frac{t}{\tau}\right)$	$\leftrightarrow \tau \text{sinc}_{\pi}(f \tau)$	

(Lathi/Ding) Fourier Transform Properties (Table 3.2)

Superposition	$g_1(t) + g_2(t)$	$G_1(f) + G_2(f)$
Scalar Multiplication	$kg(t)$	$kG(f)$
Duality	$G(t)$	$g(-f)$
Time scaling	$g(at)$	$\frac{1}{ a }G\left(\frac{f}{a}\right)$
Time shifting	$g(t - t_0)$	$G(f)e^{-j2\pi ft_0}$
Frequency shifting	$g(t)e^{j2\pi f_0 t}$	$G(f - f_0)$
Time convolution	$g_1(t) * g_2(t)$	$G_1(f)G_2(f)$
Frequency convolution	$g_1(t)g_2(t)$	$G_1(f) * G_2(f)$
Time differentiation	$\frac{d^n g(t)}{dt^n}$	$(j2\pi f)^n G(f)$
Time integration	$\int_{-\infty}^t g(x)dx$	$\frac{G(f)}{j2\pi f} + \frac{1}{2}G(0)\delta(f)$

Fourier Series

1. Trigonometric Fourier Series

$$g(t) = a_0 + a_1 \cos(\omega_0 t) + a_2 \cos(2\omega_0 t) + \dots + b_1 \sin(\omega_0 t) + b_2 \sin(2\omega_0 t) + \dots \quad (2.73a)$$

$$g(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)) \quad t_1 \leq t \leq t_1 + T_0 \quad (2.73b)$$

$$\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0}$$

$$f_0 = \frac{1}{T_0}$$

Coefficients

$$a_0 = \frac{1}{T_0} \int_{t_1}^{t_1+T_0} g(t) dt \quad n = 1, 2, 3, \dots \quad (2.76)$$

$$a_n = \frac{2}{T_0} \int_{t_1}^{t_1+T_0} g(t) \cos(n\omega_0 t) dt$$

$$b_n = \frac{2}{T_0} \int_{t_1}^{t_1+T_0} g(t) \sin(n\omega_0 t) dt \quad n = 1, 2, 3, \dots$$

2. Compact Fourier Series

$$g(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n2\pi f_0 t + \theta_n) \quad t_1 \leq t \leq t_1 + T_0 \quad (2.79)$$

$$C_0 = a_0$$

$$C_n = \sqrt{a_n^2 + b_n^2}$$

$$\theta_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right)$$

Coefficients for periodic signals with period T_0

$$a_0 = \frac{1}{T_0} \int_{T_0} g(t) dt \quad (2.83^*)$$

$$a_n = \frac{2}{T_0} \int_{T_0} g(t) \cos(n2\pi f_0 t) dt \quad n = 1, 2, 3, \dots$$

$$b_n = \frac{2}{T_0} \int_{T_0} g(t) \sin(n2\pi f_0 t) dt \quad n = 1, 2, 3, \dots$$

3. The exponential Fourier Series

$$\begin{aligned}g(t) &= \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t} \\ &= \sum_{n=-\infty}^{\infty} D_n e^{jn2\pi f_0 t}\end{aligned}\tag{2.90}$$

Coefficients (amplitudes)

$$D_n = \frac{1}{T_0} \int_{T_0} g(t) e^{-jn2\pi f_0 t} dt\tag{2.91}$$

Z-Transform

Definition of Z-transform

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] \cdot z^{-n}$$

$$X(z) = \sum_{n=0}^N x[n] \cdot (z^{-1})^n$$

(If $x[n]$ has finite support: 0 to N)

Definition of Inverse Z-transform

$$x[n] = x[n] = \frac{1}{j2\pi} \int X(z) \cdot z^{n-1} dz$$

Z-Transform common pairs

Input $x[n]$	Output $X(z)$	ROC
$\delta[n]$	$\leftrightarrow 1$	Whole z-plane
$u[n]$	$\leftrightarrow \frac{1}{1 - z^{-1}}$	$ z > 1$
$nu[n]$	$\leftrightarrow \frac{z^{-1}}{(1 - z^{-1})^2}$	$ z > 1$
$n^2u[n]$	$\leftrightarrow \frac{z^{-1}(1 + z^{-1})}{(1 - z^{-1})^3}$	$ z > 1$
$\alpha^n u[n], \quad \alpha < 1$	$\leftrightarrow \frac{1}{1 - \alpha z^{-1}}$	$ z > \alpha $
$n\alpha^n u[n], \quad \alpha < 1$	$\leftrightarrow \frac{\alpha z^{-1}}{(1 - \alpha z^{-1})^2}$	$ z > \alpha $
$\cos(\omega_0 n)u[n]$	$\leftrightarrow \frac{1 - \cos(\omega_0)z^{-1}}{1 - 2\cos(\omega_0)z^{-1} + z^{-2}}$	$ z > 1$
$\sin(\omega_0 n)u[n]$	$\leftrightarrow \frac{\sin(\omega_0)z^{-1}}{1 - 2\cos(\omega_0)z^{-1} + z^{-2}}$	$ z > 1$
$\alpha^n \cos(\omega_0 n)u[n], \alpha < 1$	$\leftrightarrow \frac{1 - \alpha \cos(\omega_0)z^{-1}}{1 - 2\alpha \cos(\omega_0)z^{-1} + \alpha^2 z^{-2}}$	$ z > 1$
$\alpha^n \sin(\omega_0 n)u[n], \alpha < 1$	$\leftrightarrow \frac{\alpha \sin(\omega_0)z^{-1}}{1 - 2\alpha \cos(\omega_0)z^{-1} + \alpha^2 z^{-2}}$	$ z > \alpha $

Z-Transform properties

1. Right shift in time of $x[n] \cdot u[n]$	$x[n] \cdot u[n - q]$	$z^{-q} \cdot X(z)$
2. Right shift of $x[n]$ ¹	$x[n - 1]$	$z^{-1} \cdot X(z) + x[-1]$
	$x[n - 2]$	$z^{-2} \cdot X(z) + x[-2] + z^{-1} \cdot x[-1]$
	$x[n - q]$	$z^{-q} \cdot X(z) + x[-q] + z^{-1} \cdot x[-q + 1] + \dots + z^{-q+1} \cdot x[-1]$ $z^{-q} \cdot X(z)$ (if $x[n] = 0$ for negative n)
3. Left shift in time	$x[n + 1]$	$z \cdot X(z) - x[0] \cdot z$
	$x[n + 2]$	$z^2 \cdot X(z) - x[0] \cdot z^2 - x[1] \cdot z$
4. Multiplication by n	$n \cdot x[n]$	$-z \cdot \frac{dX(z)}{dz}$
	$n^2 \cdot x[n]$	$z \cdot \frac{dX(z)}{dz} + z^2 \frac{d^2X(z)}{dz^2}$
5. Multiplication by a^n	$a^n \cdot x[n]$	$X\left(\frac{z}{a}\right)$
6. Multiplication by $\cos(\omega n)$ and $\sin(\omega n)$	$\cos(\omega n) \cdot x[n]$	$\frac{1}{2}[X(e^{j\omega} \cdot z) + X(e^{-j\omega} \cdot z)]$
	$\sin(\omega n) \cdot x[n]$	$\frac{j}{2}[X(e^{j\omega} \cdot z) - X(e^{-j\omega} \cdot z)]$
7. Summation $x[n] = 0$ for $n = -1, -2, -3, \dots$ i.e., a causal signal.	$\sum_{i=0}^n x[i]$	$\frac{z}{z - 1} \cdot X(z)$
8. Convolution	$x[n] * v[n]$	$X(z) \cdot V(z)$

¹ Shades of grey on this one. I saw this formula in class, so I frantically copied it down, but it is not in the book. Hopefully its right.

DTFT

(Discrete-Time Fourier Transform)

DTFT common pairs

Input $x[n]$	Output $X(e^{j\omega})$ periodic of period 2π	ROC
$\delta[n]$	$\leftrightarrow 1$	$-\pi \leq \omega < \pi$
A	$\leftrightarrow 2\pi A\delta(\omega)$	$-\pi \leq \omega < \pi$
$e^{j\omega_0 n}$	$\leftrightarrow 2\pi\delta(\omega - \omega_0)$	$-\pi \leq \omega < \pi$
$\alpha^n u[n], \alpha < 1$	$\leftrightarrow \frac{1}{1 - \alpha e^{-j\omega}}$	$-\pi \leq \omega < \pi$
$n\alpha^n u[n], \alpha < 1$	$\leftrightarrow \frac{\alpha e^{-j\omega}}{(1 - \alpha e^{-j\omega})^2}$	$-\pi \leq \omega < \pi$
$\cos(\omega_0 n)u[n]$	$\leftrightarrow \pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	$-\pi \leq \omega < \pi$
$\sin(\omega_0 n)u[n]$	$\leftrightarrow -j\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	$-\pi \leq \omega < \pi$
$\alpha^{ n }, \alpha < 1$	$\leftrightarrow \frac{1 - \alpha^2}{1 - 2\alpha\cos(\omega) + \alpha^2}$	$-\pi \leq \omega < \pi$
$p[n] = u[n + N/2] - u[n - N/2]$	$\leftrightarrow \frac{\sin(\omega(N + 1)/2)}{\sin(\omega/2)}$	$-\pi \leq \omega < \pi$
$\alpha^n \cos(\omega_0 n)u[n]$	$\leftrightarrow \frac{1 - \alpha\cos(\omega_0)e^{-j\omega}}{1 - 2\alpha\cos(\omega_0)e^{-j\omega} + \alpha^2 e^{-2j\omega}}$	$-\pi \leq \omega < \pi$
$\alpha^n \sin(\omega_0 n)u[n]$	$\leftrightarrow \frac{\alpha\sin(\omega_0)e^{-j\omega}}{1 - 2\alpha\cos(\omega_0)e^{-j\omega} + \alpha^2 e^{-2j\omega}}$	$-\pi \leq \omega < \pi$

DTFT properties

Z-transform	$x[n], X(z), z = 1 \in ROC$	$X(e^{j\omega}) = X(z) _{z=e^{j\omega}}$
Periodicity	$x[n]$	$X(e^{j\omega}) = X(e^{j(\omega+2\pi k)}), k \text{ integer}$
Linearity	$\alpha x[n] + \beta y[n]$	$\alpha X(e^{j\omega}) + \beta Y(e^{j\omega})$
Time-shifting	$x[n - N]$	$e^{-j\omega N} X(e^{j\omega})$
Frequency-shift	$x[n]e^{j\omega_0 n}$	$X(e^{j(\omega-\omega_0)})$
Convolution	$(x * y)[n]$	$X(e^{j\omega})Y(e^{j\omega})$
Multiplication	$x[n] \cdot y[n]$	$\frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$
Symmetry	$x[n]$ real-valued	$ X(e^{j\omega}) $ even function of ω $\angle X(e^{j\omega})$ odd function of ω
Parseval's relation	$\sum_{n=-\infty}^{\infty} X[n] ^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) ^2 d\omega$	